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Aggregate Effects of Imperfect Tax Enforcement

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ABSTRACT

This paper studies an economy in which the government is not able to perfectly enforce tax compliance among operating firms, and compares it with a similar economy but with perfect tax enforcement. I develop a competitive general equilibrium model where imperfect tax enforcement may affect aggregate outcomes through two mechanisms. First, it may distort firms' optimal output level as long as the probability of avoiding tax compliance is related to the firm's size. Second, poor tax enforcement may lead to a low provision of the public goods that complement firms' productivity. The results for a calibrated version of the model suggest that in economies with tax enforcement problems, aggregate output might be reduced by 12 percent. I also conclude that sizable aggregate effects can be obtained only when the public goods mechanism is at work.

Keywords: tax enforcement; public goods; informal sector; size distribution of firms

JEL Codes: E26,H41,K42,L11

1. INTRODUCTION

This paper studies an economy in which the government is not able to perfectly enforce tax compliance among operating firms, and compares it with a similar economy but with perfect tax enforcement. I develop a competitive general equilibrium model where imperfect tax enforcement may affect aggregate outcomes through two mechanisms. First, it may distort firms' optimal output level as long as the probability of avoiding tax compliance is related to the firm's size. Second, poor tax enforcement may lead to a low provision of public goods that complement firms' productivity. The results for a calibrated version of the model suggest that in economies with tax enforcement problems, aggregate output might be reduced by 12 percent and the economy's total factor productivity (TFP) by 9 percent. I also conclude that sizable aggregate effects can be obtained only when the public goods mechanism is at work.

In this paper I take the ability of a government to enforce tax compliance as an exogenous feature of the economy. I also restrict the tax system to rely completely on taxation of corporate profits. Thus the exercise here is to compare two economies that are identical except for the capacity of the government to enforce tax compliance among firms. Are aggregate equilibrium outcomes (output, TFP, average firm size, and wages) among these two economies different? If they are, then by how much? Those are the questions I try to answer in this paper.

There is empirical cross-country evidence that a strong system of legal enforcement is correlated with economic development either directly (Keefer and Knack 1995) or indirectly through the development of the financial system (La Porta et al. 1998). Here I investigate a particular channel by which legal enforcement may affect economic performance, namely, through the capacity to enforce tax compliance. In any economy taxes are necessary to raise revenue for the government, and that revenue allows the government to operate and to provide public goods and services. This paper considers the case of a benevolent government that runs a neutral tax system[1] and that maximizes aggregate consumption. However, distortions may arise because of constraints in the set of policies available to the government. In particular, the government is constrained by its tax enforcement technology.

One implication of a government's lack of enforcement capacity that has received some attention in the economic literature is the emergence of the informal sector, understood as the set of economic agents that do not comply with government regulations and taxes. A general view is that informality arrives as the response of the private sector to excessive or distortionary taxes and regulations (De Soto 1986; Maloney 2004; Rauch 1991), and in that sense it has the effect of reducing potential negative aggregate effects of government-imposed distortions on the economy. Other studies emphasize the role of the informal sector as a source of inefficiencies. Loayza (1996) develops a simple growth model in which the presence of the informal sector negatively affects growth by reducing the availability of public goods. Amaral and Quintin (2006) study a competitive model in which commitment problems are introduced in the economy due to the presence of the informal sector. Recently, De Paula and Scheinkman (2006) have developed a competitive model with two stages of production that highlights the role of value-added taxes in transmitting informality, which in turn affects firms' size decisions and the capital-labor mix. A series of case studies in *The McKinsey Quarterly* (Elstrodt, Capp, and Jones 2005; Farrell 2004; Fergie, Elstrodt, and Laboissire 2006; Lenoro, Elstrodt, and Urdapilleta 2002; Carioca, Pietracci, and Diniz 2004) suggests that the informal economy allows for the existence of less efficient firms and therefore contributes to lowering the overall productivity of the economy.

Governments that face tax compliance problems are usually not able to generate sufficient fiscal revenues, which may translate into a low provision of public services and goods. This link between enforcement capacity, the informal sector, and provision of public goods has been studied by Loayza (1996). Furthemore, the contribution of public infrastructure to output and productivity has been studied by Aschauer (1981); Lynde and Richmond (1993); Gramlich (1994); and Garcia-Mila, McGuire, and Porter (1996). In this literature it is widely accepted that public infrastructure and aggregate productivity

[1] The tax system considered here is one such that an economy with perfect tax enforcement and no public goods does not introduce any distortion into the economy.

are positively correlated, and the work by Fernald (1999) has been an important contribution to empirically establish causality. He shows a positive effect of roads on U.S. productivity. At the theoretical level tthis causality is present in the growth model of Barro and Sala-i-Martin (1992) with congestible public goods.

At the empirical level Gordon and Li (2005) have recently documented that tax revenue as a fraction of GDP is surprisingly low in developing countries compared with developed ones. I take this as an indicator of relatively low enforcement capacity in the developing world. Furthermore, it is a well-known fact that the presence of large informal sectors is an important characteristic of the developing world. Schneider and Klinglmair (2004) have estimated that the average size of the shadow economy over 1999–2000 in developing countries is 41 percent. Additionally, using cross-country data, I find the following evidence (see Appendix Tables A.1 and A.2):

- **Fact 1a:** Provision of public services is negatively correlated with the size of the informal economy.
- **Fact 1b:** Provision of public services is positively correlated with tax revenues (as a percentage of GDP).
- **Fact 2:** Provision of public services is positively correlated with GDP per capita, TFP, and output per worker.
- **Fact 3:** GDP per capita, output per worker, and TFP are negatively correlated with the size of the informal economy.

At the firm level, it is a well-established regularity that informality is negatively correlated with firm size. Moreover, the McKinsey case studies show convincing evidence that informal firms are less productive than formal ones.

The model developed in this paper emphasizes all of these empirical regularities. Building on a modified version of the Lucas span-of-control model (Lucas 1978) with a fixed labor supply, I introduce public goods into the model and, following Barro and Sala-i-Martin (1992) and Loayza (1996), consider the case of congestible public goods. The government's tax enforcement technology enters the model as a probability that a firm's profits are seized if it does not pay taxes; this probability is increasing in firm's output. In doing this I follow De Paula and Scheinkman (2006). The mechanics of the model are straightforward: When the tax enforcement technology is not perfect, some firms may find it optimal to not pay taxes and face an incentive to reduce their output level to keep a low probability of being caught by the authorities. This reduces aggregate labor demand, which calls for a lower equilibrium wage, which in turn facilitates the operation of low-productivity firms. A second mechanism works through the availability of public goods. The fraction of firms that in equilibrium decides not to pay taxes (informal sector) is high when there is a poor tax enforcement technology. In this way tax revenue and the provision of public goods as well as the overall productivity of the economy are negatively affected. As far as I know this is the first paper to consider the inclusion of public goods in a version of the Lucas (1978) span-of-control model. Another novel feature is that this paper explicitly introduces a non-distortionary tax system in a model that endogenously generates informal firms. In calibrating the model I use the observed size distribution of firms in the United States to back out the distribution of idiosyncratic productivity (or managerial talent). In order to calibrate the contribution of the public goods to the economy, I use the observed corporate tax rate by assuming it is the one that maximizes aggregate consumption.

In the next section the model is introduced. Section 3 explains the calibration strategy. Section 4 discusses the results, and in Section 5, I provide some final comments.

2. THE MODEL

This model is set up to study an economy in which the government has limited capacity to enforce corporate tax compliance. First I describe a model in which the government has full tax enforcement capacity, and then I introduce the notion of an imperfect tax enforcement technology.

Setup

I consider a one-period economy[2] populated by a mass one of households. This is a one-good economy, and each household has an endowment of $\hat{c}$ units of the good. There are two individuals in each household, a worker and an entrepreneur. All workers across households are identical in that they provide the same quality of labor services. In contrast, entrepreneurs are characterized by a parameter θ, which indicates their idiosyncratic quality in entrepreneurship or managerial talent. I use θ as an index for households and entrepreneurs since there is no other source of heterogeneity across households. θ is distributed according to a cumulative density function (CDF) $G(\theta)$ with $G(0) = 0$ and $G'(\theta) > 0$ for $\theta \in (0, \infty)$. An entrepreneur that employs l workers and k units of capital and has access to ρ units of public goods produces output equal to $y = h(\rho)\theta f(k,l)^{\gamma}$, where $f(.)$ is homogenous of degree 1 and $0 < \gamma < 1$. The parameter γ determines the degree of diminishing returns to scale of the production process; ρ represents the amount of public goods available to each production unit; and $h(.)$ is a strictly increasing and concave function. Capital is provided from outside the economy in infinite supply at a rental price r.

Within household θ the only decision maker is the entrepreneur. Her objective is to maximize the household's consumption c, which is a linear combination of both members' consumption. Entrepreneur θ faces the following decisions for given prices w and r, access to public goods ρ, and tax rate τ: the quantity of labor supply from the household's worker, whether or not she runs a production unit, and if so then how much labor l and capital k to hire. Since the worker's income can only add to household's consumption, it is optimal that the worker inelastically supply his unit of labor. The rest of the analysis takes this optimal decision as given. Thus, the problem of entrepreneur θ is

$$\max_{x \in \{0,1\}, k \geq 0, l \geq 0} c \tag{1}$$

$$\text{s.t. } c = x(1-\tau)(h(\rho)\theta f(k,l)^{\gamma} - wl - rk - \tilde{c}) + w + \hat{c},$$

where $\tilde{c}$ is a fixed cost, x is the decision to run a firm ($x = 1$ if she decides to run a firm, and 0 otherwise), and τ is the tax rate on the firm's profit. I break down Equation 1 into two problems. First, entrepreneur θ maximizes profits as if she were to run a production unit or a firm[3]:

$$\max_{k,l} (1-\tau)(h(\rho)\theta f(k,l)^{\gamma} - wl - rk - \tilde{c}) \tag{2}$$

[2] I do not introduce dynamics since here I am not interested in inter-temporal distortions. One can think of the one-period economy described here as the steady state of a dynamic model with entry and exit of firms (as in Hopenhayn 1992), where the exit rate will be given by those firms that are caught not paying taxes. Also note that because this is a static model I do not model any accumulation process either for private capital or public goods.

[3] I use these terms interchangeably.

Denote the solutions to Equation 2 as $l(\theta)$ and $k(\theta)$. I show in Appendix C that $l(\theta)$, $y(\theta) = h(\rho)\theta f(k(\theta), l(\theta))^{\gamma}$, and $\pi(\theta) = y(\theta) - wl(\theta) - rk(\theta) - \tilde{c}$ are increasing in θ and that $\pi(\theta) = (1-\tau)((1-\gamma)y(\theta) - \tilde{c})$.

Also notice that the optimal capital-labor ratio is independent of the entrepreneur's quality θ as well as of τ and $h(\rho)$.

Second, given profits $\pi(\theta)$, entrepreneur θ decides whether or not to run a firm:

$$\max_{x \in \{0,1\}} x(1-\tau)\pi(\theta) + w \tag{3}$$

Denote the solution to Equation 3 as $x(\theta)$. As long as $\pi(\theta) < 0$ it is optimal not run a firm and $x(\theta) = 0$; but if $\pi(\theta) > 0$, then $x(\theta) = 1$. Since $\pi(\theta)$ is increasing in θ, there exists a **marginal entrepreneur** θ_0 who is indifferent about running a firm. I assume $x(\theta_0) = 1$. Therefore $x(\theta) = 1$ if $\theta \in \{\theta \geq \theta_0\}$; otherwise $x(\theta) = 0$.

In this economy there is a government that can only tax firms' profits[4] and uses tax revenues to finance the provision of public goods. In particular the government announces a tax rate τ such that every firm is supposed to pay τ fraction of its profits as taxes. Consider first the perfect enforcement case, where the government has the ability to enforce tax compliance of all operating firms. Total tax revenues are then

$$T = \tau\,\Pi = \tau(1-\gamma)Y - \tau c\int_0^{\infty} x(\theta)dG(\theta), \tag{4}$$

where $\Pi = \int_0^{\infty} x(\theta)\pi(\theta)dG(\theta)$ is aggregate profit and $Y = \int_0^{\infty} x(\theta)y(\theta)dG(\theta)$ is aggregate output.

Each unit of tax revenue is transformed into one unit of a public good. I call ρ the total amount of public goods per unit of output:

$$\rho = \frac{T}{Y} \tag{5}$$

Each firm has access to ρ units of the public good,[5] and their contribution to a firm's production process is given by $h(\rho)$, where h has the following properties: $h'(\theta) > 0$ and $h''(\theta) < 0$. Thus, public goods are essential, in the sense that as ρ approaches zero, the output of every operating firm also approaches zero[6]; in addition, they are subject to decreasing returns.

Equilibrium

Given a CDF $G(\theta)$, a tax rate τ, and a rental price for capital r, an equilibrium in this economy is an allocation of capital and labor across operating plants $\{k(\theta), l(\theta)\}$ and operating decisions $\{x(\theta)\}$, a

[4] It is not my goal to study optimal taxation issues. I choose this tax system because of its neutrality. In a version of this model with no public goods and perfect tax enforcement (a version I use as a benchmark), taxing profits is fully neutral; in other words, the equilibrium is independent of the tax rate.

[5] This is the case of a public good that is rival but not excludable, and therefore it is subject to congestion. Barro and Sala-i-Martin (1992) argue that this kind of public good applies to highways and other transportation facilities, water and sewer systems, courts and domestic security.

[6] I avoid indeterminacies restricting τ to be strictly positive and less than 1.

quantity of public goods available to each operating firm ρ, and a price w all satisfying the following conditions:

1. $k(\theta)$ and $l(\theta)$ solve Equation 1 for any $\theta \in \{\theta : x(\theta) = 1\}$
2. $1 = \int_0^\infty x(\theta) l(\theta) dG(\theta)$
3. $\rho = \frac{\tau\Pi}{Y}$

Denote an equilibrium for a given tax rate τ as $\xi(\tau) = \{\{k(\theta)\}, \{l(\theta)\}, \{x(\theta)\}, \rho, w\}$.

Assumption 1. $\frac{E(\theta^z \mid \theta \geq \theta_0)}{\theta_0^z} = b$, *where* b *is a constant bigger than 1, and* z *is a finite positive number and* $\theta_0 > 0$.[7]

Proposition 1. *Given Assumption* 1 *and a tax rate* $\tau \in (0,1)$ *there is a unique equilibrium wage* w *and a unique cutoff value* θ_0 *such that* $\pi(\theta_0) = 0$, $\pi(\theta) > 0$ *for* $\theta > \theta_0$, *and* $\pi(\theta) < 0$ *for* $\theta < \theta_0$.

No-Public-Goods Case

Consider an economy with no public goods such that $h(\rho) = 1$. This will be the Lucas span-of-control model without the occupational choice margin. I show in Appendix C that if $\xi(\tilde{\tau})$ is an equilibrium for a given tax rate τ, then it is also an equilibrium for a different tax rate $\tilde{\tau}$, where τ and $\tilde{\tau}$ are in $(0,1)$. In other words, in the absence of public goods and under perfect tax enforcement, taxing profits is fully neutral. I am interested in the neutrality of the tax system due to the following. Suppose the tax system was distortive, and at the same time the government had almost no capacity to enforce tax compliance. This would be equivalent to having a very low effective tax rate and therefore almost no tax distortion in the economy. I explicitly want to avoid this positive effect on efficiency of a poor government's tax enforcement capacity.

Imperfect Tax Enforcement

Consider now the case in which the government has limited ability to enforce tax compliance. In this case an entrepreneur θ must decide whether to run a firm and comply with tax payments, or to run a firm without paying taxes,[8] or simply not to run a firm. In making that decision, firms take into account the probability of getting caught by the government if not paying taxes. If a firm is caught, then its profits are seized. The perfect enforcement case analyzed above can be understood as a particular case in which the firm always gets caught. I label a firm that decides not to pay taxes as **informal**, and conversely, one that does decide to pay taxes as **formal**.

I model the probability of getting caught by the government as increasing in output. Thus, the higher the production level of a firm, the higher the probability of getting caught. Even though I take this probability as an exogenous feature of the model, it is not difficult to justify a government that puts more effort into enforcing tax compliance among big firms than small ones. This idea has been used by De Paula and Scheinkman (2006). For convenience I focus on the probability of not getting caught, which accordingly is decreasing in output. I denote the conditional probability of not getting caught as

[7] This assumption is satisfied by a Pareto distribution with parameter α and $0 < z < \alpha$.

[8] Here I consider the case of a discrete decision, namely, paying all taxes or no taxes. I'm currently working on a continuous version where firms decide how much to pay in taxes.

$P(\text{"not getting caught"}\mid y)$ and use $P(y)$ as shorthand. $P(y)$ satisfies the following conditions: $P(0) = 1$ and $P^{'}(y) \leq 0$.

Definition 1: *A tax enforcement technology is a probability* $P(y)$ *that a firm does not get caught if it does not pay taxes.* $P(y)$ *is a better tax enforcement technology than* $\tilde{P}(y)$ *if* $P(y) \leq \tilde{P}(y)$ *and* $P(y) < \tilde{P}(y)$ *for at least some* y.

All entrepreneurs' decisions are made at the beginning of the period. At the end of the period all households will be in one of two mutually exclusive states: caught (σ_c) or not caught (σ_{nc}). Denote x_F as the decision to run a formal firm, and x_I as the decision to run an informal firm. Entrepreneur θ maximizes expected consumption and solves the following program:

$$\max_{x_I \in \{0,1\}, x_F \in \{0,1\}, k, l} P(y)c(\sigma_{cn}) + (1 - P(y))c(\sigma_c) \tag{6}$$

$$\text{s.t.} \quad c(\sigma_{nc}) = x_F(1-\tau)(y - wl - rk - \tilde{c}) + x_I(y - wl - rk - \tilde{c}) + w + \tilde{c}$$

$$c(\sigma_c) = x_F(1-\tau)(y - wl - rk - \tilde{c}) + x_I(0 - \tilde{c}) + w + \hat{c}$$

$$y = h(\rho)\theta f(k,l)^{\gamma}$$

$$x_I + x_F \neq 2$$

One can break this problem into three problems: The first is under the assumption that entrepreneur θ has decided to run a formal firm; the second is under the assumption that she has decided to run an informal firm; and the third is the decision between running a formal or an informal firm or simply not running any type of firm. For the first of these problems entrepreneur θ solves the same program as in Equation 2. I relabel $l(\theta)$ as $l_F(\theta)$ and $k(\theta)$ as $k_F(\theta)$, and define $y_F(\theta)$ and $\pi_F(\theta)$ in the obvious way.

Now take the decision of running an informal firm as given. In this case the entrepreneur θ solves the following program:

$$\max_{k,l} [P(y)(y - wl - rk) - \tilde{c}] + w + \hat{c} \tag{7}$$

$$\text{s.t.}\, y = h(\rho)\theta f(k,l)^{\gamma}$$

Define $l_I(\theta)$ and $k_I(\theta)$ as the solutions to Equation 7. Also, $y_I(\theta) = h(\rho)\theta f(k_I(\theta), l_I(\theta))$ and $\pi_I(\theta) = P(y_I(\theta))(y_I(\theta) - wl_I(\theta) - rk_I(\theta)) - \hat{c}$.

Proposition 2: *Given prices* w *and* r, *a tax rate* $\tau \in (0,1)$, *and a quantity* ρ *of public goods per firm* $y_I(\theta) \leq y_F(\theta)$, *then*

Proof: let $C(y;\theta)$ be the corresponding cost-function for an entrepreneur with quality θ. $P(y)$ is nonincreasing in y, and $y_F(\theta)$ maximizes $y - C(y,\theta)$. Therefore if $\tilde{y} > y_F(\theta)$ then:

$P(y_F(\theta))(y_F(\theta) - C(y_F(\theta),\theta)) > P(\tilde{y})(\tilde{y} - C(\tilde{y},\theta))$ q.e.d.

Now consider the decision of being a formal firm, or an informal firm, or not running a firm. Entrepreneur θ decides to be formal if $\pi_F(\theta) > \pi_I(\theta)$ and $\pi_F(\theta) \geq 0$. In this case $x_F(\theta) = 1$; otherwise, $x_F(\theta) = 0$. If $\pi_I(\theta) \geq \pi_F(\theta)$ and $\pi_I(\theta) \geq 0$, then the optimal decision is to run an informal firm, and $x_I(\theta) = 1$; otherwise, $x_I(\theta) = 0$. If $x_F(\theta) + x_I(\theta) = 0$, then it is optimal not to run a firm.

As before, the provision of public goods is fully funded by tax revenues.[9] However, tax revenues are now provided only by formal firms, such that

$$T = \tau\, \Pi_F = \tau(1-\gamma)Y_F - \tau\, \tilde{c} \int_\theta x_F(\theta) dG(\theta),$$

where $Y_F = \int_0^\infty x_F(\theta) y_F(\theta) dG(\theta)$ is the aggregate output of the formal sector, and $\Pi_F = \int_0^\infty x_F(\theta)\pi_F(\theta) dG(\theta)$ is the aggregate profit of the formal sector. ρ is the total amount of public goods per unit of aggregate output. Notice that ρ can be expressed as

$$\rho = \tau(1-\gamma)\frac{Y_F}{Y} - \frac{\tau\, \tilde{c} \int_\theta x_F(\theta) dG(\theta)}{Y}. \tag{8}$$

The first term in Equation 8 shows that as the relative importance of the formal sector declines, so does the available amount of public goods for each formal or informal firm.

Equilibriums in this economy, given a CDF $G(\theta)$, a tax enforcement technology $P(y)$, a tax rate τ, and a rental price for capital r, are an allocation of capital and labor across operating plants in the informal sector $\{k_I(\theta), l_I(\theta)\}$; an allocation of capital and labor across operating plants in the formal sector $\{k_F(\theta), l_F(\theta)\}$; operating decisions $\{x_I(\theta), x_F(\theta)\}$; a quantity of public goods available to each operating firm ρ; and a price w, all satisfying the following conditions:

1. $k_I(\theta)$ and $l_I(\theta)$ solve Equation 5 for any $\theta \in \{\theta : x_I(\theta) = 1$
2. $k_F(\theta)$ and $l_F(\theta)$ solve Equation 1 for any $\theta \in \{\theta : x_F(\theta) = 1\}$
3. $1 = \int_0^\infty x_I(\theta) l_I(\theta) dG(\theta) + \int_0^\infty x_F(\theta) l_F(\theta) dG(\theta)$
4. $\rho = \dfrac{\tau \Pi_F}{Y}$

I name an equilibrium for a given tax rate τ and a given tax enforcement technology $P(y)$ as $\xi(\tau, P) = \{\{k_I(\theta)\}, \{k_F(\theta)\}, \{l_I(\theta)\}, \{l_F(\theta)\}, \{x_I(\theta)\}, \{x_F(\theta)\}, \rho, w\}$.

The Government

In this economy the government collects revenue from taxing the profits of firms with a common tax rate τ. There is no other tax system. All tax revenues are automatically converted into public goods. The government is endowed with a tax enforcement technology $P(y)$. I will consider two cases for determining the tax rate τ. First, I consider a government that exogenously inherits a tax rate that cannot change. I label this as the **passive government case**, and it will allow me to study an economy with a suboptimal provision of public goods for a given tax enforcement technology. Second, I consider a

[9] I assume that if a firm is effectively caught, then the corresponding seized profits cannot be used to finance public goods. For simplicity, I assume those seized profits are destroyed.

benevolent government that chooses a tax rate τ in order to maximize aggregate consumption. I call this the **active government case**. Notice that the only choice available to the government is the selection of the tax rate τ while taking the competitive behavior of firms as given. The government solves the following program:

$$\max_{\tau \in (0,1)} C \tag{9}$$

subject to:

$$C = \int_0^\infty x_I(\theta)\pi_I(\theta)dG(\theta) + \int_0^\infty x_F(\theta)\pi_F(\theta)dG(\theta) + w + \hat{c}$$
$$\{\{k_I(\theta)\},\{k_F(\theta)\},\{l_I(\theta)\},\{l_F(\theta)\},\{x_I(\theta)\},\{x_F(\theta)\},\rho\} \in \xi(\tau, P).$$

3. CALIBRATION STRATEGY

I choose $f(k,l)$ to be Cobb-Douglas such that $f(k,l) = k^{\alpha} l^{1-\alpha}$ and where α captures the share of capital income out of aggregate labor and capital income. I follow the conventional choice of one-third for α. Following Atkeson and Kehoe (2005) I choose 0.85 for γ, the diminishing-returns-to-scale parameter. This implies that in an economy with perfect tax enforcement, total variable costs represent 15 percent of aggregate output. I also follow Atkeson and Kehoe (2005) in choosing 4 percent for the interest rate r.

I calibrate $G(\theta)$, the distribution of entrepreneurs' quality, by looking at the observed size distribution of firms in the U.S. economy. I assume that the U.S. government runs a perfect tax enforcement technology, and I set $h(\rho)$ as given and equal to 1. In other words, I revert to a modified version of the Lucas (1978) span-of-control model with perfect tax enforcement and no public goods, which I use as the **benchmark model**. According to Axtell (2001), the observed size distribution of firms (in terms of the number of workers) can be parameterized as a Pareto distribution with parameter δ. I show in Appendix C that if in the benchmark model θ follows a Pareto distribution with parameter $\frac{\delta}{1-\gamma}$, then the size distribution of firms also follows a Pareto distribution with parameter δ. Therefore, I use for G a Pareto CDF with parameter $\frac{\delta}{1-\gamma}$ and support $[\theta_M, \infty)$. Call L_0 the number of workers in the smallest operating firm, and $\overline{L}$ the average firm size. Using $L_0 = 2$ and the observed average firm size $\overline{L} = 21.8$, I am able to pin down δ [10] and θ_M. I set the initial household endowment $\hat{c}$ to be equal to $\tilde{c}$, so any household is at least able to pay for the fixed cost $\tilde{c}$ if the entrepeneur decides to run a firm. The fixed cost $\tilde{c}$ is arbitrarily set equal to 1. The calibration strategy is summarized in Table 1. I do not need to choose a tax rate for the benchmark model since the endogenous variables are independent of τ.

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[10] If x follows a Pareto distribution with support $[x_0, \infty)$ and parameter $\beta > 1$, then $\frac{E(x)}{x_0} = \frac{\beta}{\beta - 1}$.

Table 1. Calibration of the benchmark model

Parameters, variables	Source of numerical values, assumptions
Exogenous parameters	
$P(y)=0$	Perfect tax enforcement
$h(\rho)=1$	No public goods
$f(k,l)=k^{\alpha}l^{1-\alpha}$	Cobb-Douglas
$\alpha=\frac{1}{3}$	Capital share of output net of profits
$\gamma=0.85$	Atkeson and Kehoe (2005)
$L_0=2$	Axtell (2001)
$\overline{L}=21.8$	Axtell (2001)
$\tilde{g}=\delta\frac{L_0^{\delta}}{L^{\delta+1}}$	Observed size distribution of firms fitting a Pareto distribution
$\hat{c}=\tilde{c}$	Initial household endowment
$\tilde{c}=1$	Fixed cost, arbitrarily chosen
Calibrated parameters	
$g(\theta)=\frac{\delta}{1-\gamma}\frac{(\theta_M)^{\frac{\delta}{1-\gamma}}}{\theta^{\frac{\delta}{1-\gamma}+1}}$	Consistent with U.S. size distribution of firms $\tilde{g}(L)$
$\delta=1.10101$	$\overline{L}=\frac{\delta}{\delta-1}L_0$
$\theta_M=0.99228$	$l(\theta_0)=2$
Endogenous variables	
$\theta_0=1.5102$	
$w=1.8889$	

Next I specify the $h(\rho)$ function, which determines the importance of public goods in the model. I consider the following specific form:

$$h(\rho)=\sigma\rho^{\eta} \quad 0<\eta<1, \sigma>0.$$

Thus, public goods are essential and are subject to diminishing returns. In order to calibrate the parameters σ and η, I take the following steps:

1. I compute the equilibrium for the benchmark model calibrated to the U.S. economy. Call it ξ_{US}.

2. I add public goods to the benchmark model and assume that the government chooses a tax rate τ^* that maximizes aggregate consumption C. Call the corresponding equilibrium $\xi(\tau^*)_{US}$.
3. I find the parameters σ and η such that $\xi_{US} = \xi(\tau^*)_{US}$.

The underlying reasoning is that observable allocations and prices for the U.S. economy that allow for the calibration of the benchmark model are consistent with a provision of public goods close to its optimal level. In terms of the model, the two conditions to pin down parameters σ and η are

$$h(\rho(t^*)) = 1 \tag{10}$$

and

$$\frac{\partial C}{\partial \tau}(\tau^*) = 0 \tag{11}$$

Appendix C shows that when public goods are added to the benchmark model, then $\rho(\tau) = \tau(1-\gamma)(1-\frac{\delta-1}{\delta})$.

The ideal estimation for τ^* would be spending on public goods as a fraction of GDP in the United States. Given the lack of that information, I take two approaches. First, I rely on the observed tax rate on profits. In the United States, profits of C corporations are subject to four basic tax rates: 15, 25, 34, and 35 percent. However, any taxable income above $75,000 is subject to a tax rate of 34 percent or higher (39 percent being the highest). In the model I consider a flat tax schedule such that any unit of profit is taxed at the same rate. There is also a difference between the definition of profits in the model and what is typically considered taxable income of corporations in the United States and abroad. Taxable income of corporate profits typically reflects total revenues less payments to factors of production other than those financed by shareholders; however, in the model, payments to all factors of production are subtracted from revenues. Therefore, the observed tax rate should enter the model with an upward correction. I call ω to the fraction of capital provided by nonshareholders. Then the adjusted tax rate τ

and the observed tax rate $\tilde{\tau}$ are related according to

$$\tau(1-\gamma)y = \tilde{\tau}\,(y-(1-\alpha)\,\gamma\, y-\omega\,\alpha\,\gamma\, y)$$

or

$$\tau = \tilde{\tau}\,\frac{1-\gamma(1-\alpha(1-\omega))}{1-\gamma}.$$

The model considers the observed tax rate to be 34 percent, and for ω I use the average ratio of liabilities to the sum of equity and liabilities for U.S. firms. This average is 37.1 percent,[11] and the corresponding adjusted tax rate is 74.4 percent. At this adjusted tax rate, government expenditures in public goods reach 10.13 percent of aggregate output. This number is approximately one-third of the total government expenditures in the United States (which represents 30 percent of GDP). Second, I assume that one-fourth of government expenditures in the United States are devoted to the provision of public goods. This delivers a more conservative estimate for the adjusted tax rate, 55 percent. Table 2 summarizes the calibration strategy for $h(\rho;\eta)$.

[11] I would like to thank Daisuke Miyakawa, who computed this ratio for me using Compustat.

Table 2. Calibration of public goods function $h(\rho)$

$h(\rho\,;\sigma,\eta) = \sigma\rho^{\eta}$			
Parameters, variables			**Source of numerical values, assumptions**
		τ^* based on observed tax rate	
$\tilde{\tau}$	=	34%	observed tax rate
ω	=	0.371	average $\dfrac{\text{liabilities}}{\text{equity} + \text{liabilities}}$, (Compustat)
τ^*	=	74.4%	adjusted tax rate
σ	=	1.31142	calibrated parameter
η	=	0.11843	calibrated parameter
ρ	=	10.13%	ratio of public goods expenditures to output
		τ^* based on expenditures in public goods	
ρ	=	7.5%	assuming expenditures in public goods $= \dfrac{1}{4} G_{US}$
τ^*	=	55%	adjusted tax rate
σ	=	1.24275	calibrated parameter
η	=	0.08387	calibrated parameter

Finally, the model introduces the probability that a firm does not get caught if it does not pay taxes, $P(y)$, using the following functional form:

$$P(y) = \begin{cases} 1 & \text{if } y \le \bar{y} \\ \left(\dfrac{\underline{y}}{y}\right)^{\phi} & \text{if } y > \bar{y} \end{cases}$$

with $0 < \phi < 1$. Figure 1 shows the shape of this function. I set the parameter $\underline{y}$ to be slightly above the output level of the smallest firm in the benchmark model.[12] This functional form provides a closed form for $y_I(\theta)$:

$$y_I(\theta) = \begin{cases} y_F(\theta) & \text{if } \theta \le \underline{\theta} \\ \left(\dfrac{1-\phi}{1-\gamma\phi}\right)^{\frac{\gamma}{1-\gamma}} y_F(\theta) & \text{if } \underline{\theta} < \theta \end{cases}$$

where

$$y_F(\theta) = \left(\gamma\left(\frac{1-\alpha}{w}\right)^{1-\alpha}\left(\frac{\alpha}{r}\right)^{\alpha}\right)^{\frac{\gamma}{1-\gamma}} (h(\rho)\theta)^{\frac{1}{1-\gamma}}$$

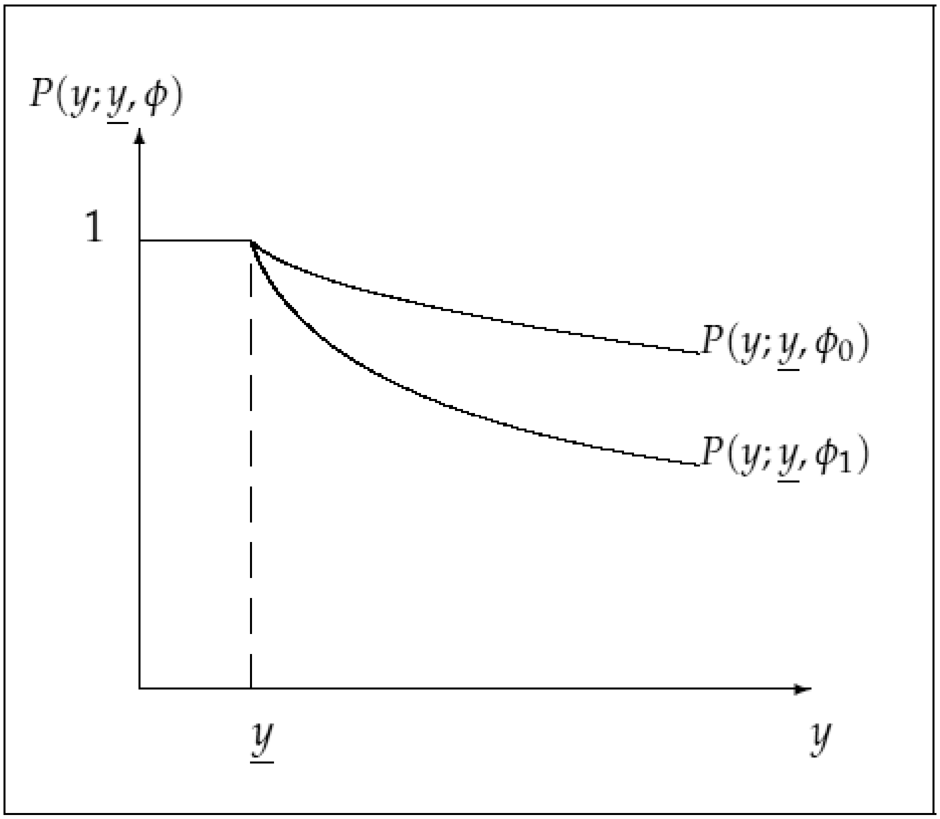
and

$$\underline{\theta} = y_F^{-1}(\underline{y}).$$

[12] It is 0.5 above. This addition is needed to prevent some very small firms from becoming formal instead of informal.

It is important to notice that the ratio between $y_I(\theta)$ and $y_F(\theta)$ is decreasing in ϕ for $\phi \in [0,1]$.[13] In the case of a government fully incapable of enforcing tax compliance ($\phi = 0$), then informal firms do not need to hide through reductions in their output levels and will choose the same output as if they were formal. Also notice that the tax rate τ does not affect the ratio $\frac{y_I(\theta)}{y_F(\theta)}$. I leave the parameter ϕ free, which allows me to introduce different levels of tax enforcement capacity into the model. As ϕ moves from 0 to 1, the tax enforcement technology improves.

Figure 1. Tax enforcement technology $P(y)$



[13] This is for $\theta \in (\underline{\theta}, \theta_1)$.

4. RESULTS

In the previous section I showed the choices for the functional forms and parameters implied by the model, except for the parameter ϕ of the tax enforcement technology $P(y)$. An extreme case is $\phi = 0$, which implies no tax enforcement at all, so that any firm can avoid paying taxes and not get caught. As ϕ increases, conditional on output level, the probability that a firm gets caught is increased,[14] and therefore according to Definition 1 the tax enforcement technology improves. Since I am interested in the potential effects of a poor tax enforcement technology, I solve the model for different values of ϕ and investigate for differences in aggregate output, measured TFP, equilibrium wage rate, the size of the informal sector, and the average size of firms. The discipline on how far ϕ can increase is given by the model's prediction of the size of the informal economy. I use as a benchmark the perfect enforcement case in which all firms comply with tax payments.

I consider two cases for the government's behavior. One case assumes a benevolent government that, given a tax enforcement technology, chooses a tax rate $\tau \in [0,1)$ such that aggregate consumption C is maximized. I call this the **active government case**. I also consider a **passive government case** in which the government exogenously inherits a tax rate that cannot adjust.

In addition I study the effects of an imperfect tax enforcement technology in the absence of public goods. In the model, as the tax enforcement technology worsens, ceteris paribus, more firms become informal, and therefore tax revenues and the provision of public goods per unit of output decreases. This in turn reduces productivity of all of the operating firms. In the absence of public goods, this effect is not present. However, given a poor tax enforcement technology, the incentives for a firm to become informal are still at work. A subset of firms may optimally choose not to pay taxes and may reduce their input demands relative to the perfect enforcement case. This puts into work a general equilibrium effect that distorts the allocation of resources across firms. I call this the **no-public-goods case**. Since a benevolent or active government would like to minimize the effects of its imperfect tax enforcement, it will trivially choose a zero tax rate; therefore, I only consider the no-public-goods case under the assumption of a passive government.

Table 3 summarizes the results for different cases. In particular, the public goods function $h(.)$ has been calibrated so that the optimal provision of public goods is 10.13 percent of aggregate output and is consistent with $\tau = 74.4\%$. As exhibited in the top panel of Table 3, I consider a tax enforcement technology $P(y)$ with parameter $\phi = 0.15$. To interpret this number I compute the probability that a firm in the 90th percentile of the size distribution of firms[15] gets caught if it does not pay taxes. This probability is 26.1 percent. At this level of tax enforcement capacity and assuming a benevolent government, the model predicts an output level 12.1 percent below the perfect enforcement case, and an informal sector that accounts for 30.3 percent of aggregate output. Moreover, observed TFP and wages are 8.6 and 12.7 percent below the benchmark case. According to Schneider (2004), more than half of a sample of 144 countries, mainly developing economies, exhibit similar or higher levels of informal output. At this level of tax enforcement capacity, the government finds it optimal to set the tax rate at 46 percent, well below the optimal tax rate of the benchmark case. Reducing the tax rate positively affects the consumption of households running formal firms. Also, ceteris paribus, it reduces tax collection and therefore the provision of public firms. However, this effect is compensated by a reduction in the number of firms that choose to become informal such that tax collection and provision of public goods increase.

[14] This holds for any output level bigger than $\underline{y}$.

[15] This corresponds to the size distribution of operating firms under the benchmark case.

Table 3. Aggregate effects of imperfect tax enforcement

	Optimal $\tau = 74.4\%$ in benchmark model				
	Aggregate output (Y)	TFP	Wage	θ_0	Informal output (%)
	Perfect enforcement				
Benchmark model	3.333	1.361	1.888	1.510	0.00
	Imperfect enforcement				
	$(\phi = 0.15)$				
- Active government ($\tau = 0.46$)	2.931	1.244	1.648	1.527	30.29
- Passive government	2.929	1.245	1.636	1.519	56.57
No public goods	3.317	1.361	1.853	1.493	56.57
	$(\phi = 0.20)$				
- Active government ($\tau = 0.49$)	3.006	1.267	1.689	1.521	23.53
- Passive government	3.049	1.283	1.700	1.508	44.54
No public goods	3.314	1.361	1.848	1.491	44.54
	$(\phi = 0.30)$				
- Active government ($\tau = 0.53$)	3.103	1.296	1.745	1.514	13.58
- Passive government	3.178	1.322	1.771	1.497	28.15
No public goods	3.307	1.360	1.843	1.489	28.15
ϕ:	0.15	0.20	0.30		
Probability a firm in 90th percentile gets caught	26.13%	33.22%	45.43%		

In addition, I find a selection effect that reduces the number of operating firms by 7.8 percent relative to the benchmark. These are firms that would be operating if it were not for the fact that the government operates a poor tax enforcement technology. Two countereffects are at work: First, wages that are lower than in the benchmark will lead to entry of low-productivity firms (negative selection). Second, scarce provision of public goods negatively affects the productivity of all firms and will lead to the exit of low-productivity firms (positive selection). The empirical results show that the second effect dominates. When I solve the model for the same tax enforcement level, assuming a passive government, the effects on output, TFP, and wages are very similar. However, the size of the informal sector is much higher at 56.6 percent.[16] Also, the selection effect is lower, accounting for an exit of 4.2 percent of firms relative to the benchmark model.

[16] Countries at this level of informal sector size are Nigeria, United Republic of Tanzania, Zimbabwe, Peru, Azerbaijan, Panama, Bolivia, and Georgia

When considering the no-public-goods case, the negative effects of the poor tax enforcement technology over output, TFP, and wages are drastically reduced. Aggregate output now is less than 1 percent below the benchmark, the effect on TFP is negligible, and wages are lower by 2 percent. However, now there is a negative selection effect instead of a positive one. The size of this effect is 8.67 percent, which means this is the fraction of additional operating firms relative to the benchmark, which would not have operated with perfect enforcement. Since in this case the effect of public goods provision on productivity is not present, the only mechanism at work comes through lower wages. What I conclude from these results is that in the model, sizable effects of a poor tax enforcement technology arise only under the presence of the public goods mechanism. In a broader sense, this points to the idea that distortions that mainly affect the left tail of the size distribution of firms (small firms) will have limited aggregate effects unless there is a more direct feedback effect over the entire distribution beyond the standard general equilibrium effects.

In the middle and bottom panels of Table 3, I consider less severe tax enforcement inefficiencies. The model predicts lower levels of informality (24 percent and 14 percent) and, as expected, smaller reductions—albeit still significant—in aggregate output, TFP, and wages. Equilibrium aggregate output is reduced by 10 percent and 7 percent, respectively. Also notice that in the absence of public goods, a low tax enforcement capacity can deliver equilibrium output, TFP, and wages that are higher than in an economy with a better enforcement technology. That occurs because at low levels of enforcement, informal firms have almost no incentive to reduce their output levels relative to the output they would have chosen under perfect tax enforcement. At the same time, the effect of informal firms on provision of public goods and overall productivity is not present.

Table 4 shows some predictions of the model for firm size and distribution of the labor force across formal and informal firms. Since by construction the tax enforcement capacity of the government is higher among large firms, the model predicts that informal firms belong to left tail of the size distribution of firms, while formal ones belong to the right tail. Also, the model predicts a discontinuity in the size distribution of firms such that no firms of intermediate size will be observed. This feature of the model matches qualitatively with the "missing middle," in which there is a smaller number of medium-size firms in less developed countries relative to what it is observed in developed countries. Tybout (2000) has documented the missing middle for the manufacturing sector.

In the model, the range of the missing middle and the "mean missing size" decrease with tax enforcement capacity. Similar predictions apply to the informal sector. When considering a low tax-enforcement technology (top panel of Table 4), the informal sector is populated by firms with less than 108 workers. And the missing middle is in the range of firms with 108 to 128 workers. Tybout (2000) shows evidence of a missing middle in the range of 20 to 100 workers for a country like Mexico, where the size of the informal sector is 30 percent. Therefore, it seems that the average missing middle the model predicts is too high. I claim this is in part because there is another well-studied fact of informal (small) firms in least developed countries that the model does not incorporate, namely, that informal firms are significantly more labor-intensive than formal ones (Amaral and Quintin 2006; Tybout 2000).

One can think that this empirical evidence can be matched to the model by adding size-dependent financial frictions (as in Amaral and Quintin 2006) or a probability of detection increasing in capital (as in De Paula and Scheinkman 2006). I leave these modifications as candidates for future improvements of the model and for now simply claim that these modifications will at least reduce the lower bound of the missing middle predicted by the model. As one considers better tax enforcement technologies (middle and bottom panels of Table 4), the model's prediction for the missing middle looks closer to the empirical evidence. The model also does well in predicting a negative correlation between the fraction of the labor force in the informal sector and the level of tax enforcement capacity.

Table 4. Labor force and firm size ($\tau = 74.4\%$ in benchmark)

	Mean	Informal sector				Formal
	size	LF(%)	Min	Mean	Max	Min
			Perfect enforcement			
Benchmark model	21.8	0.00	--	--	--	2.00
			Imperfect enforcement			
			$(\phi = 0.15)$			
Active government	23.68	29.76	2.29	7.13	108	128.73
Passive government	22.78	55.94	2.31	12.74	17,395	20,705
No public goods	20.11	55.94	2.04	11.25	15,361	18,284
			$(\phi = 0.20)$			
Active government	22.96	22.89	2.24	5.42	41	53
Passive government	21.63	43.65	2.22	9.45	1,550	1,982
No public goods	19.91	43.65	2.04	8.70	1,426	1,823
			$(\phi = 0.30)$			
Active government	22.34	14.45	2.18	3.56	12.80	18.68
Passive government	20.49	26.95	2.13	5.57	110.02	166.67
No public goods	19.69	26.95	2.05	5.35	105.74	160.18

ϕ:	0.15	0.20	0.30
Probability a firm in 90th percentile gets caught	26.13%	33.22%	45.43%

In Appendix Tables B.1 and B.2, I show results under the assumption that aggregate consumption is maximized when expenditures on public goods represent 7.5 percent of the aggregate output in the benchmark economy. This implies a tax rate on profits of 55 percent. In the case where the optimal tax rate equals 74.4 percent, the aggregate effects of imperfect tax enforcement are reduced. When analyzing a low tax enforcement capacity $(\phi = 0.15)$, the result is that aggregate output is lower by 6.3 percent, observed TFP by 4.4 percent, and wages by 6.9 percent, where all figures are relative to the benchmark model; and the size of the informal economy reaches 24.38 percent. At this level of lack of enforcement, an active government will set the tax rate at 39 percent, 16 points lower than the corresponding optimal tax rate for the benchmark economy. I confirm also that without public goods in the model, it is very difficult to obtain sizable effects on output, TFP, and wages. In the absence of public goods, the reduction on equilibrium aggregate output generated is lower than 0.5 percent.

5. FINAL COMMENTS

In this paper I investigate the economic implications of technological differences in the government side of the economy. In particular I study economies that have access to the same set of resources and productive technologies, but where governments are endowed with different technologies to enforce tax compliance. My implicit assumption is that while private technologies can freely flow across economies, this is not the case for some government-related technologies.

The paper shows that a government's lack of tax enforcement capacity may have implications for aggregate output, TFP, wages, and the size of the informal economy. I mainly exploit two mechanisms through which government's lack of tax enforcement capacity distorts the economy. First, provided that the government operates a tax enforcement technology that is more efficient in detecting tax evasion by large firms than small ones, firms face an incentive to reduce their optimal output level. Second, a poor tax enforcement technology may lead to a fiscal problem and low provision of productive public goods. The results for a calibrated version of the model suggest that in economies where the government has limited capacity to enforce tax compliance, aggregate output is lower by 12 percent and TFP by 9 percent relative to an economy where the government can perfectly enforce tax compliance. Moreover, poor tax enforcement suggests an informal sector that accounts for 30 percent or more of aggregate output.

Based on the numerical predictions of the model I also conclude that sizable aggregate effects can be reached only when the public goods mechanism is at work. This suggests that any competitive equilibrium model that introduces a friction distorting mainly economic decisions on the left tail of the size distribution of firms (small firms) will not be able to deliver sizable aggregate effects unless it also incorporates an externality or feedback effect (beyond standard general equilibrium effects) on the right tail of the distribution (large firms). This is because distorting decisions of small and medium-size firms is equivalent to a distortion that only binds a small share of the economic activity.

APPENDIX A: EMPIRICAL EVIDENCE

Table A.1. Facts 1a and 1b: Cross-country evidence (Correlation, Number of Observation)

	Informal sector (% GDP)	Tax revenues (% GDP)	Tax compliance
Paved roads per capita	-0.5154*	0.3934*	0.3726*
	136	106	43
Paved roads per km2	-0.4448*	0.1906*	0.3222*
	136	106	43
Days to resolve a debt dispute	0.2396*	-0.2328*	-0.4517*
	143	95	43
Cost to resolve debt dispute	0.1998*	-0.2185*	-0.3609*
	143	95	43

Data Source: Schneider (2004), Caselli (2004), World Development Indicators
Ross Levine, "Financial Structure and Economic Growth: A Cross-Country Comparison of Banks, Markets, and Development"
http://www.econ.brown.edu/fac/Ross_Levine/Publications.htmData
Note: *10% significance level; GPD (1+informal size) was also used.

Table A.2. Facts 2 and 3: Cross-country evidence (Correlations, Number of observations)

	GDP per capita	Y/L	TFP	TFP H adjusted
Paved roads per capita	0.7302*	0.7806*	0.7059*	0.5694*
	162	100	99	90
Paved roads per km^2	0.5321*	0.5936*	0.5453*	0.4896*
	162	100	99	90
Days to resolve a debt dispute	-0.2841*	-0.3119*	-0.2641*	-0.1159
	142	100	99	89
Cost to resolve debt dispute	-0.3531*	-0.3944*	-0.4327*	-0.4295*
	142	100	99	89
Informal sector (% GDP)	-0.6889*	-0.7396*	-0.7103*	-0.6041*
	135	96	95	85

Data Source: Schneider (2004), Caselli (2004), World Development Indicators
Ross Levine, "Financial Structure and Economic Growth: A Cross-Country Comparison of Banks, Markets, and Development"
http://www.econ.brown.edu/fac/Ross_Levine/Publications.htmData
Note: * 10% significance level; GPD(1+informal size) was also used; H: Human capital

APPENDIX B: NUMERICAL RESULTS

Table B.1. Effects of imperfect tax enforcement

Optimal $\tau = 55\%$ in Benchmark Model					
	Aggregate output (Y)	TFP	Wage	θ_D	Informal output (%)
	Perfect enforcement				
Benchmark model	3.333	1.361	1.888	1.510	0.00
	Imperfect enforcement				
	$(\phi = 0.15)$				
- Active government ($\tau = 0.39$)	3.122	1.301	1.758	1.516	24.38
- Passive government	3.174	1.318	1.781	1.508	37.99
No public goods	3.322	1.361	1.864	1.499	37.99
	$(\phi = 0.20)$				
- Active government ($\tau = 0.41$)	3.170	1.315	1.785	1.512	17.62
- Passive government	3.229	1.334	1.812	1.504	28.03
No public goods	3.320	1.361	1.863	1.498	28.03
	$(\phi = 0.30)$				
- Active government ($\tau = 0.44$)	3.232	1.333	1.824	1.510	7.77
- Passive government	3.289	1.351	1.848	1.500	14.83
No public goods	3.319	1.360	1.865	1.499	14.83

ϕ:	0.15	0.20	0.30
Probability a firm in 90th percentile gets caught	26.13%	33.22%	45.43%

Table B.2. Labor force and firm size ($\tau = 55\%$ in benchmark)

	Mean size	Informal sector LF (%)	Informal sector Min	Informal sector Mean	Informal sector Max	Formal Min
	Perfect enforcement					
Benchmark model	21.8	0.00	--	--	--	2.00
	Imperfect enforcement					
	$(\phi = 0.15)$					
Active government	23.17	32.65	2.20	7.63	141.74	158.25
Passive government	21.61	37.40	2.12	8.11	358.30	426.47
No public goods	20.65	37.40	2.02	7.74	342.32	407.45
	$(\phi = 0.20)$					
Active government	22.43	23.92	2.15	5.54	41.41	49.29
Passive government	21.17	27.32	2.09	5.87	77.72	99.34
No public goods	20.58	27.32	2.03	5.70	75.57	96.60
	$(\phi = 0.30)$					
Active government	21.83	7.38	2.07	2.30	4.07	6.17
Passive government	20.85	14.10	2.04	3.24	11.72	17.76
No public goods	20.66	14.10	2.03	3.21	11.62	17.60

ϕ:	0.15	0.20	0.30
Probability a firm in 90th percentile gets caught	26.13%	33.22%	45.43%

APPENDIX C: STATEMENTS AND PROOFS OF PROPOSITIONS

C.1. Output is Increasing in θ

By assumption $f(k,l)$ is homogenous of degree 1 in k and l. Therefore it can be expressed as $f(k,l) = l\,\mathrm{f(k)}$ where $\mathrm{k} = \frac{k}{l}$.

Cost function:

$$\min_{k,l}\; wl + rk$$

$$\text{s.t. } y = h(\rho)\theta(l\,\mathrm{f(k)})^{\gamma}$$

First-order conditions:

$$w = \lambda h(\rho)\theta\gamma(lf(k))^{\gamma-1}(f(k) - f_k(k)k) \tag{12}$$

$$r = \lambda h(\rho)\theta\gamma(lf(k))^{\gamma-1} f_k(k) \tag{13}$$

Combine Equations 12 and 13:

$$\frac{\mathrm{f(k)}}{\mathrm{f_k(k)}} - \mathrm{k} - \frac{w}{r} = 0.$$

Given standard production function conditions $\mathrm{f_k(k)} > 0$ and $\mathrm{f_{kk}(k)} < 0$ and using the implicit function theorem, k is increasing in $\frac{w}{r}$

$$\mathrm{k} = \mathrm{k}\left(\frac{w}{r}\right).$$

Labor demand for a given output level y:

$$l = \left(\frac{y}{h(\rho)\theta}\right)^{\frac{1}{\gamma}} \frac{1}{\mathrm{f(k)}} \tag{14}$$

Express total variable cost $wl + rk$ as $l(w + rk)$. Use Equation 14 and get the cost function:

$$C(y) = \left(\frac{y}{h(\rho)\theta}\right)^{\frac{1}{\gamma}} \frac{w + r\,\mathrm{k}}{\mathrm{f(k)}} \tag{15}$$

Use the first-order condition:

$$1 = C'(y) \tag{16}$$

to get the optimal output decision for a given θ:

$$y(\theta) = \left(\frac{\gamma\,\mathrm{f(k)}}{w + r\,\mathrm{k}}\right)^{\frac{\gamma}{1-\gamma}} (h(\rho)\theta)^{\frac{1}{1-\gamma}}.$$

Since $0 < \gamma < 1$, $y(\theta)$ is increasing in θ.

An alternative expression for $y(\theta)$ is $y(\theta) = \left(\frac{\gamma \, \mathrm{f_k(k)}}{r}\right)^{\frac{\gamma}{1-\gamma}} (h(\rho)\theta)^{\frac{1}{1-\gamma}}$.

C.2 Labor Demand is Increasing in θ

Equation 16 implies:

$$y = \frac{1}{\gamma}\left(\frac{y}{h(\rho)\theta}\right)^{\frac{1}{\gamma}} \frac{w + r\,\mathrm{k}}{\mathrm{f(k)}} \quad (17)$$

Plug Equation 17 and $y(\theta)$ into Equation 14 and get optimal labor decision:

$$l(\theta) = \left(\frac{1}{w + r\,\mathrm{k}}\right)\gamma y(\theta) \quad (18)$$

Since $y(\theta)$ is increasing in θ then $l(\theta)$ is also increasing in θ.

C.3. Profits

From Equation 18:

$$l(\theta)(w + r\,\mathrm{k}) = \gamma \; y(\theta).$$

Therefore,

$$\pi(\theta) = (1-\tau)((1-\gamma)y(\theta) - \tilde{c}).$$

Since $y(\theta)$ is increasing in θ, then $\pi(\theta)$ is also increasing in θ.

C.4. Proof of Proposition 1

$\pi^{'}(\theta) > 0$ and $\pi(0) < 0 \;\Rightarrow\; \exists$ a unique θ_0 such that $\pi(\theta_0) = 0$ and:

$$x(\theta) = \begin{cases} 0 & \text{if } \; 0 \le \theta < \theta_0 \\ 1 & \text{if } \; \theta_0 \le \theta < \infty \end{cases}$$

Zero-profit condition:
Condition $\pi(\theta_0) = 0$ implies

$$y(\theta_0) = \frac{\tilde{c}}{1-\gamma} \quad (19)$$

and

$$\left(\frac{\gamma \, \mathrm{f_k(k)}}{r}\right)^{\frac{\gamma}{1-\gamma}} (h(\rho)\theta_0)^{\frac{1}{1-\gamma}} = \frac{\tilde{c}}{1-\gamma}. \quad (20)$$

Use Equation 19 and Assumption 1 to express ρ as:

$$\rho = \tau(1-\gamma)(1-\frac{1}{b}),$$

where b is a constant greater than 1.

Apply the implicit function theorem to Equation 20 and get

$$w = w_1(\theta_0, h(\rho)), \tag{21}$$

which is an increasing function in θ_0. By inspection it is easy to see that as $\theta_0 \to 0$ also $w \to 0$.

Labor market equilibrium:
Express

$$1 = \int_0^\infty x(\theta) l(\theta) dG(\theta)$$

as

$$1 = l(\theta_0) \int_{\theta_0}^\infty \left(\frac{\theta}{\theta_0}\right)^{\frac{1}{1-\gamma}} dG(\theta),$$

Plug $l(\theta)$ and Equation 19 into this expression:

$$1 = \left(\frac{1}{w + r\ \mathrm{k}}\right) \frac{\gamma\ \tilde{c}}{1-\gamma} \int_{\theta_0}^\infty \left(\frac{\theta}{\theta_0}\right)^{\frac{1}{1-\gamma}} dG(\theta) \tag{22}$$

Multiply and divide Equation 22 by $1-G(\theta_0)$ and use Assumption 1:

$$1 = \left(\frac{1}{w + r\ \mathrm{k}}\right) b(1 - G(\theta_0)) \tag{23}$$

Apply the implicit function theorem to Equation 23 and get

$$w = w_2(\theta_0), \tag{24}$$

which is a decreasing function of θ_0. By inspection it is easy to see that as $\theta_0 \to 0$ then $w \to \infty$.

Equations 21 and 24 intersect once. Then there is a unique θ_0 that satisfies

$$w_1(\theta_0) = w_2(\theta_0) > 0.$$

C.5. Benchmark Model's Equilibrium is Independent of τ

In the benchmark model, $h(\rho) = 1$. Therefore Equation 21 becomes independent of τ. Equation 24 is also independent of τ. Therefore, any pair $\{w, \theta_0\}$ that satisfies Equations 21 and 24 is independent of τ.

C.6. Pareto Distribution of θ

Let's assume that employment l follows a Pareto distribution with parameters δ and L_0. The corresponding density function is

$$f(l) = \frac{\delta\, l_0^{\delta}}{l^{\delta+1}} \quad l \geq l_0 \, .$$

In the standard model, employment l is related to θ according to:

$$l(\theta) = A\theta^{\frac{1}{1-\gamma}} \quad A > 0 \, .$$

Then, I can infer a density function for θ according to:

$$g(\theta) = f(l(\theta))\left|\frac{\partial l(\theta)}{\partial \theta}\right| \quad \text{and } l_0 = l(\theta_0) \, .$$

After replacing the terms:

$$g(\theta) = \frac{\frac{\delta}{1-\gamma}\theta_0^{\frac{\delta}{1-\gamma}}}{\theta^{\frac{\delta}{1-\gamma}+1}} \, ,$$

which is a Pareto density function with parameters θ_0 and $\frac{\delta}{1-\gamma}$. A property of the Pareto distribution is that one can interpret $g(\theta)$ as the density function of θ conditional on $\theta > \theta_0$. And by the same property I can call

$$g(\theta) = \frac{\frac{\delta}{1-\gamma}\theta_M^{\frac{\delta}{1-\gamma}}}{\theta^{\frac{\delta}{1-\gamma}+1}} \quad \theta_M < \theta_0$$

the unconditional density function of θ with parameters θ_M and $\frac{\delta}{1-\gamma}$.

C.7. Public Goods

Express ρ as

$$\rho = \tau(1-\gamma) - \frac{\tilde{c}\,\tau}{y(\theta_0)\int_{\theta_0}^{\infty}(\frac{\theta}{\theta_0})^{\frac{1}{1-\gamma}} \frac{dG(\theta)}{(1-G(\theta_0))}} \, . \tag{25}$$

Since $G(\theta)$ is the CDF of a Pareto distribution with parameters θ_M and $\frac{\delta}{1-\gamma}$, then

$$\begin{aligned}
\int_{\theta_0}^{\infty}\left(\frac{\theta}{\theta_0}\right)^{\frac{1}{1-\gamma}}\frac{dG(\theta)}{1-G(\theta_0)} &= \int_{\theta_0}^{\infty}\left(\frac{\theta}{\theta_0}\right)^{\frac{1}{1-\gamma}}\frac{\frac{\delta}{1-\gamma}\theta_0^{\frac{\delta}{1-\gamma}}}{\theta^{\frac{\delta}{1-\gamma}+1}}d\theta \\
&= \frac{\left(\frac{\delta}{1-\gamma}\right)}{\left(\frac{\delta-1}{1-\gamma}\right)}\int_{\theta_0}^{\infty}\frac{\frac{\delta-1}{1-\gamma}\theta_0^{\frac{\delta-1}{1-\gamma}}}{\theta^{\frac{\delta-1}{1-\gamma}+1}}d\theta \\
&= \frac{\delta}{\delta-1}.
\end{aligned}$$

Plug this expression and $y(\theta_0) = \frac{c}{1-\gamma}$ into Equation 25 to get

$$\rho = \tau(1-\gamma)(1-\frac{\delta-1}{\delta}) \qquad (26)$$

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